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LETTER TO THE EDITOR

Critical couplings of mixed spin-$\frac{1}{2}$–spin-S Ising model: a free-fermion approximation

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Abstract. A free-fermion approximation procedure is applied to the mixed spin-$\frac{1}{2}$-spin-S Ising models. Our results for the critical coupling K_c are in very satisfactory agreement with the high-temperature series results of Yousif and Bowers.

The mixed spin-$\frac{1}{2}$-spin-S models, considered by Yousif and Bowers (1984), have less translational symmetry than their 'single spin' counterparts and are well adapted for the study of a certain type of ferrimagnetism. The reduced Hamiltonian of this model takes the form

$$-\beta H = K \sum_{ij} \sigma_i S_j. \tag{1}$$

The underlying lattice is composed of two interpenetrating sublattices, one being occupied by 'spins' σ_i of magnitude $\frac{1}{2}$ whilst the alternate one is occupied by 'spins' S_j of magnitude S. The σ_i take the values $\pm\frac{1}{2}$ and the S_j the values $-S, -S+1, \ldots, S$, where S has one of the usual integral or odd half-integral values. The summation (1) involves all pairs of nearest-neighbour sites in the lattice.

In this letter, we apply a free-fermion approximation procedure to this model. The critical coupling K_c given by this method is in very satisfactory agreement with high-temperature series results obtained by Yousif and Bowers (1984).

First, we map the model (1) into the eight-vertex model by summing all the spin S in the lattice. This leads to the following weights of the eight-vertex model:

$$\begin{aligned} &w_1 = w(++++) && w_2 = w(-+-+)\\ &w_3 = w(--++) && w_4 = w(+--+)\\ &w_5 = w(---+) && w_6 = w(-+--)\\ &w_7 = w(+---) && w_8 = w(--+-) \end{aligned} \tag{2}$$

with

$$\begin{aligned} w(\sigma_1, \sigma_2, \sigma_3, \sigma_4) &= \sum_S \exp(K(\sigma_1+\sigma_2+\sigma_3+\sigma_4)S)\\ &= \begin{cases} 1 + 2\sum_{n=1}^{S} \cosh nK(\sigma_1+\sigma_2+\sigma_3+\sigma_4) & S = \text{integral}\\ 2\sum_{n=1}^{2S} \cosh \frac{1}{2}nK(\sigma_1+\sigma_2+\sigma_3+\sigma_4) & S = \text{odd half-integral.} \end{cases} \end{aligned} \tag{3}$$

Table 1. Estimates of K_c for mixed spin-$\frac{1}{2}$-spin-S model with $S=\frac{1}{2}$, 1, $\frac{3}{2}$, 5, 10, 100.

S	$\frac{1}{2}$	1	$\frac{3}{2}$	5	10	100
K_c (our method)	$2\ln(1+\sqrt{2})$	1.022	0.7352	0.2560	0.1335	0.0172
Δ/w_1^2	0	0.0165	0.020	0.0256	0.0261	0.0287
K_c (series[a])	1.764 ± 0.070	1.025 ± 0.007	0.736 ± 0.015	0.2564 ± 0.0080	0.1336 ± 0.0044	0.0140 ± 0.0006

[a] Yousif and Bowers (1984).

The eight-vertex model has been solved approximately by Fan and Wu (1969) when $\Delta/w_m^2 \ll 1$ where $w_m = \max(w_1, w_2, w_3\ w_4)$, and

$$\Delta = w_1 w_2 + w_3 w_4 - w_5 w_6 - w_7 w_8. \qquad (4)$$

They gave the critical condition for this model

$$w_1 = \bar{w}_2 + w_3 + w_4 \qquad (5)$$

where

$$\bar{w}_2 = w_2 - \Delta/w_1. \qquad (6)$$

We have calculated the critical coupling K_c by equation (5) for the cases $S=\frac{1}{2}$, 1, $\frac{3}{2}$, 5, 10, 100 (see table 1). The series results are also given in table 1 for comparison. In the case of $S=\frac{1}{2}$, our results become exact and in the remaining cases our results are surprisingly good compared with series results. Since the order of correction Δ/w_1^2 increases with the value of S, the error becomes larger for the large value of S (see table 1).

References

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